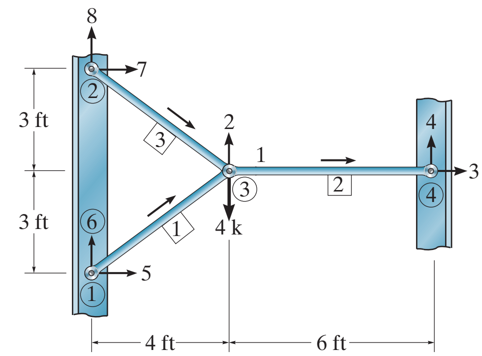
14–1. Determine the stiffness matrix **K** for the assembly. Take $A = 0.5\text{ in}^2$ and $E = 29(10^3)$ ksi for each member.



Member 1: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-0}{5} = 0.6$

$$\mathbf{k}_1 = \frac{AE}{60}\begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix}$$

Member 2: $\lambda_x = \dfrac{10-4}{6} = 1; \qquad \lambda_y = \dfrac{3-3}{6} = 0$

$$\mathbf{k}_2 = \frac{AE}{72}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

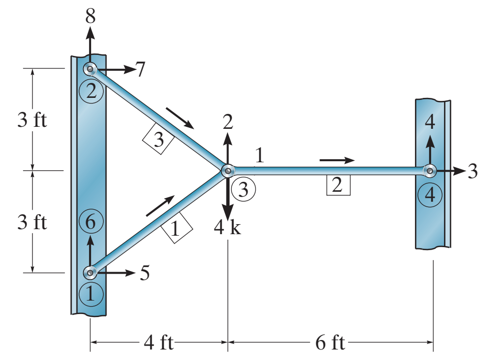
Member 3: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-6}{5} = -0.6$

$$\mathbf{k}_3 = \frac{AE}{60}\begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix}$$

Assembly stiffness matrix: $\mathrm{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$

$$\mathbf{K} = \begin{bmatrix} 510.72 & 0 & -201.39 & 0 & -154.67 & -116 & -154.67 & 116 \\ 0 & 174 & 0 & 0 & -116 & -87.0 & 116 & -87.0 \\ -201.39 & 0 & 201.39 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -154.67 & -116 & 0 & 0 & 154.67 & 116 & 0 & 0 \\ -116 & -87.0 & 0 & 0 & 116 & 87.0 & 0 & 0 \\ -154.67 & 116 & 0 & 0 & 0 & 0 & 154.67 & -116 \\ 116 & -87.0 & 0 & 0 & 0 & 0 & -116 & 87.0 \end{bmatrix} \qquad \textbf{Ans.}$$

14–2. Determine the horizontal and vertical displacements at joint ③ of the assembly in Prob. 14–1.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \qquad \mathbf{Q}_k = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Use the assembly stiffness matrix of Prob. 14–1 and applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 0 \\ -4 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 510.72 & 0 & -201.39 & 0 & -154.67 & -116 & -154.67 & 116 \\ 0 & 174 & 0 & 0 & -116 & -87.0 & 116 & -87.0 \\ -201.39 & 0 & 201.39 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -154.67 & -116 & 0 & 0 & 154.67 & 116 & 0 & 0 \\ -116 & -87.0 & 0 & 0 & 116 & 87.0 & 0 & 0 \\ -154.67 & 116 & 0 & 0 & 0 & 0 & 154.67 & -116 \\ 116 & -87.0 & 0 & 0 & 0 & -0 & -116 & 87.0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Partition matrix

$$0 = 510.72(D_1) + 0(D_2)$$

$$-4 = 0(D_1) + 174(D_2)$$

Solving

$$D_1 = 0$$

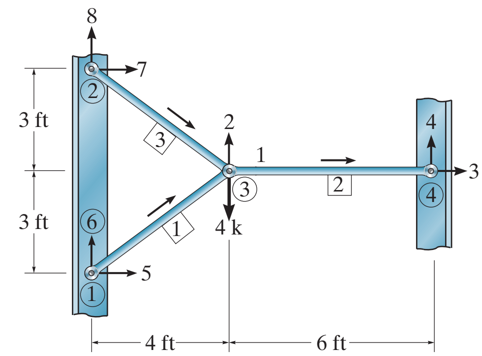
$$D_2 = -0.022990 \text{ in.}$$

Thus,

$$D_1 = 0 \qquad \textbf{Ans.}$$

$$D_2 = -0.0230 \text{ in.} \qquad \textbf{Ans.}$$

14–3. Determine the force in each member of the assembly in Prob. 14–1.



From Prob. 14–2.

$$D_1 = D_3 = D_4 = D_5 = D_6 = D_7 = D_8 = 0 \quad D_2 = -0.02299$$

To calculate force in each member, use Eq. 14–23.

$$q_F = \frac{AE}{L}\begin{bmatrix}-\lambda_x & -\lambda_y & \lambda_x & \lambda_y\end{bmatrix}\begin{bmatrix}D_{N_x}\\ D_{N_y}\\ D_{F_x}\\ D_{F_y}\end{bmatrix}$$

Member 1: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-0}{5} = 0.6$

$$q_1 = \frac{AE}{L}\begin{bmatrix}-0.8 & -0.6 & 0.8 & 0.6\end{bmatrix}\begin{bmatrix}0\\ 0\\ 0\\ -0.02299\end{bmatrix}$$

$$q_1 = \frac{0.5(29(10^3))}{60}(0.6)(-0.02299) = -3.33\text{ k} = 3.33\text{ k (C)} \qquad \textbf{Ans.}$$

Member 2: $\lambda_x = \dfrac{10-4}{6} = 1; \qquad \lambda_y = \dfrac{3-3}{6} = 0$

$$q_2 = \frac{AE}{L}\begin{bmatrix}-1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\ -0.02299\\ 0\\ 0\end{bmatrix}$$

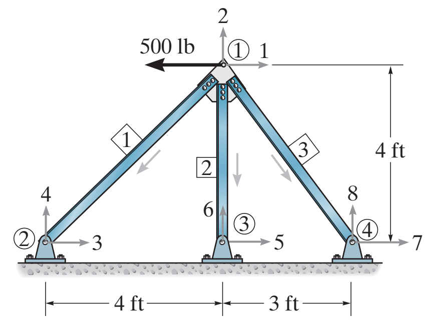
$$q_2 = 0 \qquad \textbf{Ans.}$$

Member 3: $\lambda_x = \dfrac{4-0}{5} = 0.8; \qquad \lambda_y = \dfrac{3-6}{5} = -0.6$

$$q_3 = \frac{AE}{L}\begin{bmatrix}-0.8 & 0.6 & 0.8 & -0.6\end{bmatrix}\begin{bmatrix}0\\ 0\\ 0\\ -0.02299\end{bmatrix}$$

$$q_3 = \frac{0.5(29(10^3))}{60}(-0.6)(-0.02299) = 3.33\text{ k (T)} \qquad \textbf{Ans.}$$

***14–4.** Determine the stiffness matrix **K** for the truss. Take $A = 0.75 \text{ in}^2$, $E = 29(10^3)$ ksi.



Member 1: $\lambda_x = \dfrac{0 - 4}{\sqrt{32}} = -0.7071 \qquad \lambda_y = \dfrac{0 - 4}{\sqrt{32}} = -0.7071$

$$\mathbf{k}_1 = AE\begin{bmatrix} 0.08839 & 0.08839 & -0.08839 & -0.08839 \\ 0.08839 & 0.08839 & -0.08839 & -0.08839 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 \end{bmatrix}$$

Member 2: $\lambda_x = \dfrac{4 - 4}{4} = 0 \qquad \lambda_y = \dfrac{0 - 4}{4} = -1$

$$\mathbf{k}_2 = AE\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & -0.25 \\ 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0.25 \end{bmatrix}$$

Member 3: $\lambda_x = \dfrac{7 - 4}{5} = 0.6 \qquad \lambda_y = \dfrac{0 - 4}{5} = -0.8$

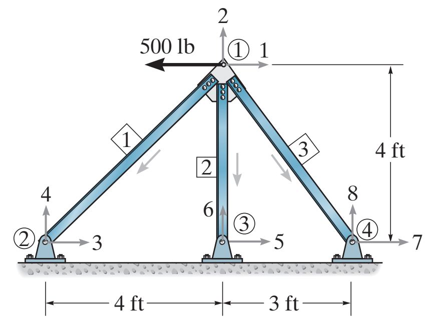
$$\mathbf{k}_3 = AE\begin{bmatrix} 0.072 & -0.096 & -0.072 & 0.096 \\ -0.096 & 0.128 & 0.096 & -0.128 \\ -0.072 & 0.096 & 0.072 & -0.096 \\ 0.096 & -0.128 & -0.096 & 0.128 \end{bmatrix}$$

Structure stiffness matrix

$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$

$$\mathbf{K} = AE\begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.128 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.128 & 0 & 0 & 0 & 0 & -0.096 & 0.128 \end{bmatrix} \quad \textbf{Ans.}$$

14–5. Determine the horizontal displacement of joint ① and the force in member $\boxed{2}$. Take $A = 0.75\ \text{in}^2$, $E = 29(10^3)$ ksi.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_k = \begin{bmatrix} -500 \\ 0 \end{bmatrix}$$

Use the structure stiffness matrix of Prob. 14–4 and applying $\mathbf{Q} = \mathbf{KD}$. We have

$$\begin{bmatrix} -500 \\ 0 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = AE \begin{bmatrix} 0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096 \\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.128 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096 \\ 0.096 & -0.128 & 0 & 0 & 0 & 0 & -0.096 & 0.128 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Partition matrix

$$\begin{bmatrix} -500 \\ 0 \end{bmatrix} = AE \begin{bmatrix} 0.16039 & -0.00761 \\ -0.00761 & 0.46639 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-500 = AE(0.16039D_1 - 0.00761D_2) \qquad (1)$$

$$0 = AE(-0.00761D_1 + 0.46639D_2) \qquad (2)$$

Solving Eq. (1) and (2) yields:

$$D_1 = \frac{-3119.82}{AE} = \frac{-3119.85(12\ \text{in./ft})}{0.75\ \text{in}^2(26)(10^6)\ \text{lb/in}^2} = -0.00172\ \text{in.}$$ **Ans.**

$$D_2 = \frac{-50.917}{AE}$$

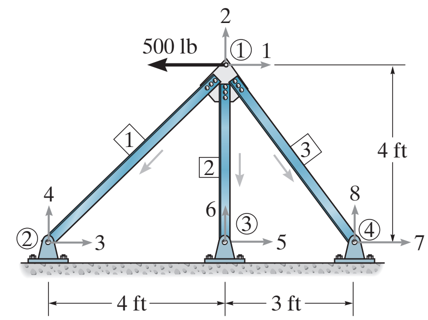
For member 2

$\lambda_x = 0, \qquad \lambda_y = -1, \qquad L = 4$ ft

$$q_2 = \frac{AE}{4}\begin{bmatrix} 0 & 1 & 0 & -1 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} -3119.85 \\ -50.917 \\ 0 \\ 0 \end{bmatrix}$$

$$= -12.73\ \text{lb} = 12.7\ \text{lb (C)}$$ **Ans.**

14–6. Determine the force in member $\boxed{2}$ if its temperature is increased by 100°F. Take $A = 0.75\ \text{in}^2$, $E = 29(10^3)$ ksi, $\alpha = 6.5(10^{-6})/°\text{F}$.



$$\begin{bmatrix}(Q_1)_0\\(Q_2)_0\\(Q_3)_0\\(Q_4)_0\end{bmatrix} = AE(6.5)(10^{-6})(+100)\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} = AE\begin{bmatrix}0\\-650\\0\\650\end{bmatrix}(10^{-4})$$

Use the structure stiffness matrix of Prob. 14–4.

$$\begin{bmatrix}-500\\0\\Q_3\\Q_4\\Q_5\\Q_6\\Q_7\\Q_8\end{bmatrix} = AE\begin{bmatrix}0.16039 & -0.00761 & -0.08839 & -0.08839 & 0 & 0 & -0.072 & 0.096\\ -0.00761 & 0.46639 & -0.08839 & -0.08839 & 0 & -0.25 & 0.096 & -0.1280\\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0\\ -0.08839 & -0.08839 & 0.08839 & 0.08839 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -0.25 & 0 & 0 & 0 & 0.25 & 0 & 0\\ -0.072 & 0.096 & 0 & 0 & 0 & 0 & 0.072 & -0.096\\ 0.096 & -0.1280 & 0 & 0 & 0 & 0 & -0.096 & 0.1280\end{bmatrix}\begin{bmatrix}D_1\\D_2\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

$$+\ AE\begin{bmatrix}0\\-650\\0\\650\\0\\0\\0\\0\end{bmatrix}(10^{-6})$$

$$\frac{-500}{(0.75)(29)(10^6)} = 0.16039D_1 - 0.00761D_2 + 0$$

$$0 = -0.00761D_1 + 0.46639D_2 - 650(10^{-6})$$

Solving yields

$$D_1 = -77.837(10^{-6})\text{ ft}$$

$$D_2 = 1392.427(106^{-6})\text{ ft}$$

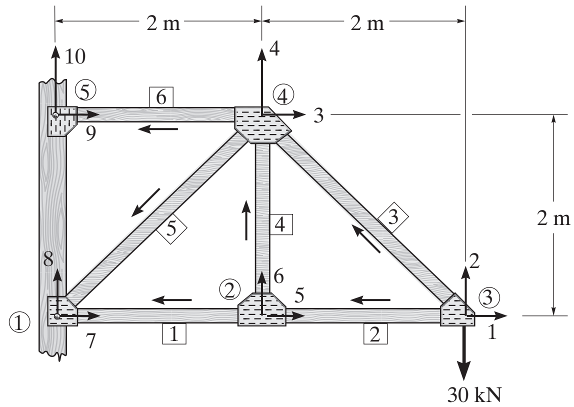
For member 2

$$\lambda_x = 0, \qquad \lambda_y = -1, \qquad L = 4\text{ ft}$$

$$q_2 = \frac{0.75(29)(10^6)}{4}[0 \quad 1 \quad 0 \quad -1]\begin{bmatrix}-77.837\\1392.427\\0\\0\end{bmatrix}(10^{-6}) - 0.75(29)(10^6)(6.5)(10^{-6})(100)$$

$$= 7571.32 - 14\,137.5 = -6566.18\text{ lb} = 6.57\text{ k(C)}$$ **Ans.**

14–7. Determine the stiffness matrix **K** for the truss. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



The origin of the global coordinate system will be set at joint ①.

For member $\boxed{1}$, $L = 2$ m. $\quad \lambda_x = \dfrac{0-2}{2} = -1 \qquad \lambda_y = \dfrac{0-0}{2} = 0$

$$\mathbf{k}_1 = \frac{0.0015[200(10^9)]}{2}\begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} 150(10^6) & 0 & -150(10^6) & 0 \\ 0 & 0 & 0 & 0 \\ -150(10^6) & 0 & 150(10^6) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

For member $\boxed{2}$, $L = 2$ m. $\quad \lambda_x = \dfrac{2-4}{2} = -1 \qquad \lambda_y = \dfrac{0-0}{2} = 0$

$$\mathbf{k}_2 = \frac{0.0015[200(10^9)]}{2}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 150(10^6) & 0 & -150(10^6) & 0 \\ 0 & 0 & 0 & 0 \\ -150(10^6) & 0 & 150(10^6) & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

For member $\boxed{3}$, $L = 2\sqrt{2}$m. $\quad \lambda_x = \dfrac{2-4}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{2} \qquad \lambda_y = \dfrac{2-0}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$$\mathbf{k}_3 = \frac{0.0015[200(10^9)]}{2\sqrt{2}}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 53.033(10^6) & -53.033(10^6) & -53.033(10^6) & 53.033(10^6) \\ -53.033(10^6) & 53.033(10^6) & 53.033(10^6) & -53.033(10^6) \\ -53.033(10^6) & 53.033(10^6) & 53.033(10^6) & -53.033(10^6) \\ 53.033(10^6) & -53.033(10^6) & -53.033(10^6) & 53.033(10^6) \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

14–7. Continued

For member $\boxed{4}$, $L = 2$ m.

$$\lambda_x = \frac{2-2}{2} = 0 \qquad \lambda_y = \frac{2-0}{2} = 1$$

$$\mathbf{k}_4 = \frac{0.0015[200(10^9)]}{2} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 150(10^6) & 0 & -150(10^6) \\ 0 & 0 & 0 & 0 \\ 0 & -150(10^6) & 0 & 150(10^6) \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$

For member $\boxed{5}$, $L = 2\sqrt{2}$ m.

$$\lambda_x = \frac{0-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \lambda_y = \frac{2-0}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\mathbf{k}_5 = \frac{0.0015[200(10^9)]}{2\sqrt{2}} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 7 \\ 8 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 3 & 4 & 7 & 8 \end{array} \\ \begin{bmatrix} 53.033(10^6) & 53.033(10^6) & -53.033(10^6) & -53.033(10^6) \\ -53.033(10^6) & 53.033(10^6) & -53.033(10^6) & -53.033(10^6) \\ -53.033(10^6) & -53.033(10^6) & 53.033(10^6) & 53.033(10^6) \\ 53.033(10^6) & -53.033(10^6) & 53.033(10^6) & 53.033(10^6) \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 7 \\ 8 \end{array}$$

For member $\boxed{6}$, $L = 2$ m.

$$\lambda_x = \frac{0-2}{2} = -1 \qquad \lambda_y = \frac{2-2}{2} = 0$$

$$\mathbf{k}_6 = \frac{0.0015[200(10^9)]}{2} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 9 & 10 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 9 \\ 10 \end{array}$$

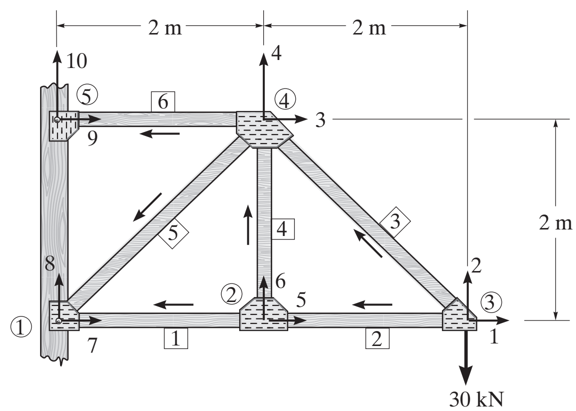
$$= \begin{array}{c} \begin{array}{cccc} 3 & 4 & 9 & 10 \end{array} \\ \begin{bmatrix} 150(10^6) & 0 & -150(10^6) & 0 \\ 0 & 0 & 0 & 0 \\ -150(10^6) & 0 & 150(10^6) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 9 \\ 10 \end{array}$$

14–7. Continued

Structure stiffness matrix is a 10 × 10 matrix since the highest code number is 10. Thus,

$$
\begin{array}{c}
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
\end{array}
\\
\begin{bmatrix}
203.033 & -53.033 & -53.033 & 53.033 & -150 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 53.033 & -53.033 & 0 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 256.066 & 0 & 0 & 0 & -53.033 & -53.033 & -150 & 0 \\
53.033 & -53.033 & 0 & 256.066 & 0 & -150 & -53.033 & -53.033 & 0 & 0 \\
-150 & 0 & 0 & 0 & 300 & 0 & -150 & 0 & 0 & 0 \\
0 & 0 & 0 & -150 & 0 & 150 & 0 & 0 & 0 & 0 \\
0 & 0 & -53.033 & -53.033 & -150 & 0 & 203.033 & 53.033 & 0 & 0 \\
0 & 0 & -53.033 & -53.033 & 0 & 0 & 53.033 & 53.033 & 0 & 0 \\
0 & 0 & -150 & 0 & 0 & 0 & 0 & 0 & 150 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array}
(10^6) \qquad \textbf{Ans.}
$$

***14–8.** Determine the vertical displacement at joint ② and the force in member $\boxed{5}$. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa.



Here,

$$
Q_k = \begin{bmatrix} 0 \\ -30(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\qquad
D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 7 \\ 8 \\ 9 \\ 10 \end{array}
$$

Then, applying $\mathbf{Q} = \mathbf{KD}$

$$
\begin{bmatrix} 0 \\ -30(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ \hdashline Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \end{bmatrix}
=
\left[\begin{array}{cccccccccc}
203.033 & -53.033 & -53.033 & 53.033 & -150 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 53.033 & -53.033 & 0 & 0 & 0 & 0 & 0 & 0 \\
-53.033 & 53.033 & 256.066 & 0 & 0 & 0 & -53.033 & -53.033 & -150 & 0 \\
53.033 & -53.033 & 0 & 256.066 & 0 & -150 & -53.033 & -53.033 & 0 & 0 \\
-150 & 0 & 0 & 0 & 300 & 0 & -150 & 0 & 0 & 0 \\
0 & 0 & 0 & -150 & 0 & 150 & 0 & 0 & 0 & 0 \\ \hdashline
0 & 0 & -53.033 & -53.033 & -150 & 0 & 203.033 & 53.033 & 0 & 0 \\
0 & 0 & -53.033 & -53.033 & 0 & 0 & 53.033 & 53.033 & 0 & 0 \\
0 & 0 & -150 & 0 & 0 & 0 & 0 & 0 & 150 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
(10^6)
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ \hdashline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

14–8. Continued

From the matrix partition, $Q_k = K_{11}D_u + K_{12}D_k$ is given by

$$\begin{bmatrix} 0 \\ -30(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 203.033 & -53.033 & -53.033 & 53.033 & -150 & 0 \\ -53.033 & 53.033 & 53.033 & -53.033 & 0 & 0 \\ -53.033 & 53.033 & 256.066 & 0 & 0 & 0 \\ -53.033 & -53.033 & 0 & 256.066 & 0 & -150 \\ -150 & 0 & 0 & 0 & 300 & 0 \\ 0 & 0 & 0 & -150 & 0 & 150 \end{bmatrix}(10^6)\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = [203.033D_1 - 53.033D_2 - 53.033D_3 + 53.033D_4 - 150D_5](10^6) \qquad (1)$$

$$-30(10^3) = [-53.033D_1 + 53.033D_2 + 53.033D_3 - 53.033D_4](10^6) \qquad (2)$$

$$0 = [-53.033D_1 + 53.033D_2 + 256.066D_3](10^6) \qquad (3)$$

$$0 = [53.033D_1 - 53.033D_2 + 256.066D_4 - 150D_6](10^6) \qquad (4)$$

$$0 = [-150D_4 + 300D_5](10^6) \qquad (5)$$

$$0 = [-150D_4 + 150D_6](10^6) \qquad (6)$$

Solving Eqs (1) to (6),

$D_1 = -0.0004$ m $\quad D_2 = -0.0023314$ m $\quad D_3 = 0.0004$ m $\quad D_4 = -0.00096569$ m

$D_5 = -0.0002$ m $\quad D_6 = 0.00096569$ m $\quad = 0.000966$ m **Ans.**

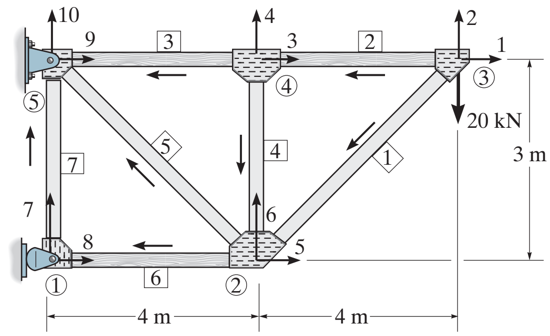
Force in member $\boxed{5}$. Here $\lambda_x = -\dfrac{\sqrt{2}}{2}$, $\lambda_y = -\dfrac{\sqrt{2}}{2}$ and $L = 2\sqrt{2}$ m

Applying Eqs 14–23,

$$(q_5)_F = \frac{0.0015[200(10^9)]}{2\sqrt{2}}\begin{bmatrix} \dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} & -\dfrac{\sqrt{2}}{2} & -\dfrac{\sqrt{2}}{2} \end{bmatrix}\begin{array}{c} \begin{bmatrix} \\ \\ 0 \\ 0 \end{bmatrix} \end{array}\begin{array}{c} 3 \\ 4 \\ 7 \\ 8 \end{array}$$

$$= -42.4 \text{ kN}$$ **Ans.**

14–9. Determine the stiffness matrix **K** for the truss. Take $A = 0.0015 \text{ m}^2$ and $E = 200$ GPa for each member.



The origin of the global coordinate system will be set at joint ①.

For member $\boxed{1}$, $L = 5$ m, $\lambda_x = \dfrac{4-8}{5} = -0.8$ and $\lambda_y = \dfrac{0-3}{5} = -0.6$

$$\mathbf{k}_1 = \frac{0.0015[200(10^9)]}{5}\begin{array}{c}\begin{array}{cccc}1 & 2 & 5 & 6\end{array}\\ \begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix}\end{array}\begin{array}{c}\\1\\2\\5\\6\end{array}$$

$$= \begin{array}{c}\begin{array}{cccc}1 & 2 & 5 & 6\end{array}\\ \begin{bmatrix} 38.4 & 28.8 & -38.4 & -28.8 \\ 28.8 & 21.6 & -28.8 & -21.6 \\ -38.4 & -28.8 & 38.4 & 28.8 \\ -28.8 & -21.6 & 28.8 & 21.6 \end{bmatrix}\end{array}\begin{array}{c}\\1\\2\\5\\6\end{array}(10^6)$$

For member $\boxed{2}$, $L = 4$ m, $\lambda_x = \dfrac{4-8}{4} = -1$ and $\lambda_y = \dfrac{3-3}{0} = 0$

$$\mathbf{k}_2 = \frac{0.0015[200(10^9)]}{4}\begin{array}{c}\begin{array}{cccc}1 & 2 & 3 & 4\end{array}\\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\end{array}\begin{array}{c}\\1\\2\\3\\4\end{array}$$

$$= \begin{array}{c}\begin{array}{cccc}1 & 2 & 3 & 4\end{array}\\ \begin{bmatrix} 75 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 \\ -75 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\end{array}\begin{array}{c}\\1\\2\\3\\4\end{array}(10^6)$$

14–9. Continued

For member $\boxed{3}$, $L = 4$ m, $\quad \lambda_x = \dfrac{0-4}{4} = -1$ and $\lambda_y = \dfrac{3-3}{4} = 0$

$$\mathbf{k}_3 = \frac{0.0015[200(10^9)]}{4} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 9 \\ 10 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 3 & 4 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} 75 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 \\ -75 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 9 \\ 10 \end{array} (10^6)$$

For member $\boxed{4}$, $L = 3$ m, $\quad \lambda_x = \dfrac{4-4}{3} = 0$ and $\lambda_y = \dfrac{0-3}{3} = -1$

$$\mathbf{k}_4 = \frac{0.0015[200(10^9)]}{3} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & -100 \\ 0 & 0 & 0 & 0 \\ 0 & -100 & 0 & 100 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array} (10^6)$$

For member $\boxed{5}$, $L = 5$ m, $\quad \lambda_x = \dfrac{0-4}{5} = -0.8$ and $\lambda_y = \dfrac{3-0}{5} = 0.6$

$$\mathbf{k}_5 = \frac{0.0015[200(10^9)]}{5} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 9 \\ 10 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 5 & 6 & 9 & 10 \end{array} \\ \left[\begin{array}{cccc} 38.4 & -28.8 & -38.4 & 28.8 \\ -28.8 & 21.6 & 28.8 & -21.6 \\ -38.4 & 28.8 & 38.4 & -28.8 \\ 28.8 & -21.6 & -28.8 & 21.6 \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 9 \\ 10 \end{array} (10^6)$$

14–9. Continued

For member $\boxed{6}$, $L = 4$ m, $\lambda_x = \dfrac{0-4}{4} = -1$ and $\lambda_y = \dfrac{0-0}{4} = 0$

$$\mathbf{k}_6 = \frac{0.0015[200(10^9)]}{4}\begin{array}{c c} \begin{array}{cccc} 5 & 6 & 8 & 7 \end{array} & \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 5 \\ 6 \\ 8 \\ 7 \end{array} \end{array}$$

$$= \begin{array}{c c} \begin{array}{cccc} 5 & 6 & 8 & 7 \end{array} & \\ \begin{bmatrix} 75 & 0 & -75 & 0 \\ 0 & 0 & 0 & 0 \\ -75 & 0 & 75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} 5 \\ 6 \\ 8 \\ 7 \end{array} \end{array} (10^6)$$

For member $\boxed{7}$, $L = 3$ m, $\lambda_x = \dfrac{0-0}{3} = 0$ and $\lambda_y = \dfrac{3-0}{3} = 1$

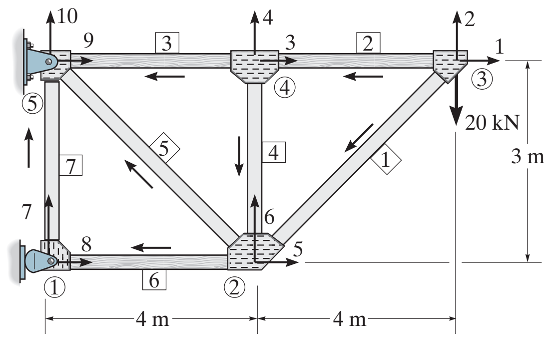
$$\mathbf{k}_7 = \frac{0.0015[200(10^9)]}{3}\begin{array}{c c} \begin{array}{cccc} 8 & 7 & 9 & 10 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} & \begin{array}{c} 8 \\ 7 \\ 9 \\ 10 \end{array} \end{array}$$

$$= \begin{array}{c c} \begin{array}{cccc} 8 & 7 & 9 & 10 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & -100 \\ 0 & 0 & 0 & 0 \\ 0 & -100 & 0 & 100 \end{bmatrix} & \begin{array}{c} 8 \\ 7 \\ 9 \\ 10 \end{array} \end{array} (10^6)$$

Structure stiffness matrix is a 10×10 matrix since the highest code number is 10. Thus,

$$\begin{array}{c c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} & \\ \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 & 0 & 0 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 & 0 & 0 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 & 0 & 0 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 & -75 & -38.4 & 28.8 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 & 0 & 28.8 & -21.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -100 \\ 0 & 0 & 0 & 0 & -75 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & -75 & 0 & -38.4 & 28.8 & 0 & 0 & 113.4 & -28.8 \\ 0 & 0 & 0 & 0 & 28.8 & -21.6 & -100 & 0 & -28.8 & 121.6 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \end{array} (10^6) \qquad \textbf{Ans.}$$

14–10. Determine the force in member $\boxed{5}$. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



Here,

$$Q_k = \begin{array}{c} \begin{bmatrix} 0 \\ -20(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \qquad D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 8 \\ 9 \\ 10 \end{array}$$

Then applying $\mathbf{Q} = \mathbf{KD}$

$$\left[\begin{array}{c} 0 \\ -20(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline Q \\ Q_9 \\ Q_{10} \end{array}\right] = \left[\begin{array}{ccccccc:ccc} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 & 0 & 0 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 & 0 & 0 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 & 0 & 0 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 & -75 & -38.4 & 28.8 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 & 0 & 28.8 & -21.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -100 \\ \hdashline 0 & 0 & 0 & 0 & -75 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & -75 & 0 & -38.4 & 28.8 & 0 & 0 & 113.4 & -28.8 \\ 0 & 0 & 0 & 0 & 28.8 & -21.6 & -100 & 0 & -28.8 & 121.6 \end{array}\right](10^6) \left[\begin{array}{c} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ \hdashline 0 \\ 0 \\ 0 \end{array}\right]$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$ is given by

$$\begin{bmatrix} 0 \\ -20(10^3) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & 28.8 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}(10^6) \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = (113.4D_1 + 28.8D_2 - 75D_3 - 384D_5 - 28.8D_6)(10^6) \qquad (1)$$

$$-20(10^3) = (28.8D_1 + 21.6D_2 - 28.8D_5 - 21.6D_6(10^6)) \qquad (2)$$

$$0 = (-75D_1 + 150D_3)(10^6) \qquad (3)$$

$$0 = (100D_4 - 100D_6)(10^6) \qquad (4)$$

$$0 = (-38.4D_1 - 28.8D_2 + 151.8D_5)(10^6) \qquad (5)$$

$$0 = (-28.8D_1 - 21.6D_2 + 100D_4 + 143.2D_6)(10^6) \qquad (6)$$

$$0 = (100D_7)(10^6) \qquad (7)$$

14–10. Continued

Solving Eqs (1) to (7)

$D_1 = 0.000711 \quad D_2 = -0.00470 \quad D_3 = 0.000356 \quad D_4 = -0.00187$

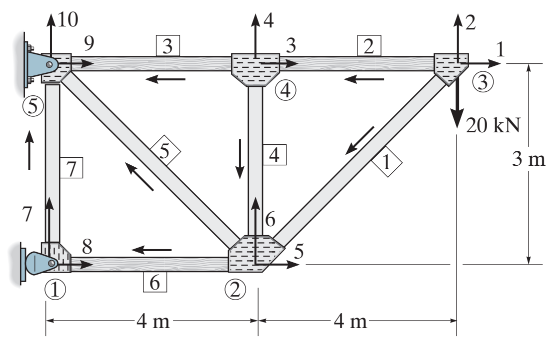
$D_5 = -0.000711 \quad D_6 = -0.00187 \quad D_7 = 0$

Force in member $\boxed{5}$. Here $L = 5\ m$, $\lambda_x = -0.8$ and $\lambda_y = 0.6$.

$$(q_5)_F = \frac{0.0015[200(10^9)]}{5}\begin{bmatrix} 0.8 & -0.6 & -0.8 & 0.6 \end{bmatrix} \begin{array}{c} \begin{bmatrix} -0.000711 \\ -0.00187 \\ 0 \\ 0 \end{bmatrix} \end{array}\begin{array}{c} 5 \\ 6 \\ 9 \\ 10 \end{array}$$

$= 33.3\text{ kN}$ **Ans.**

14–11. Determine the vertical displacement of node ② if member $\boxed{6}$ was 10 mm too long before it was fitted into the truss. For the solution, remove the 20-k load. Take $A = 0.0015\ \text{m}^2$ and $E = 200$ GPa for each member.



For member $\boxed{6}$, $L = 4$ m, $\lambda_x = -1$, $\lambda_y = 0$ and $\Delta_L = 0.01$ m. Thus,

$$\begin{bmatrix} (Q_5)_0 \\ (Q_6)_0 \\ (Q_7)_0 \\ (Q_8)_0 \end{bmatrix} = \frac{0.00015[200(10^9)](0.001)}{4}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.75 \\ 0 \\ 0.75 \\ 0 \end{bmatrix}\begin{array}{c} 5 \\ 6 \\ 8 \\ 7 \end{array}(10^6)$$

Also

$$Q_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \quad \text{and } D_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} 8 \\ 9 \\ 10 \end{array}$$

Applying $\mathbf{Q} = \mathbf{KD} + \mathbf{Q}_0$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_8 \\ Q_9 \\ Q_{10} \end{bmatrix} = \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 & 0 & 0 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 & 0 & 0 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 & 0 & -75 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 & 0 & 0 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 & -75 & -38.4 & 28.8 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 & 0 & 28.8 & -21.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -100 \\ 0 & 0 & 0 & 0 & -75 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & -75 & 0 & -38.4 & 28.8 & 0 & 0 & 113.4 & -28.8 \\ 0 & 0 & 0 & 0 & 28.8 & -21.6 & -100 & 0 & -28.8 & 121.6 \end{bmatrix}(10^6)\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.75 \\ 0 \\ 0 \\ 0.75 \\ 0 \\ 0 \end{bmatrix}(10^6)$$

14–11. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k + (\mathbf{Q}_k)_0$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 113.4 & 28.8 & -75 & 0 & -38.4 & -28.8 & 0 \\ 28.8 & 21.6 & 0 & 0 & -28.8 & -21.6 & 0 \\ -75 & 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & -100 & 0 \\ -38.4 & -28.8 & 0 & 0 & 151.8 & 0 & 0 \\ -28.8 & -21.6 & 0 & -100 & 0 & 143.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} (10^6) \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.75 \\ 0 \\ 0 \end{bmatrix} (10^6)$$

Expanding this matrix equality,

$0 = (113.4D_1 + 28.8D_2 - 75D_3 - 38.4D_5 - 28.8D_6)(10^6)$ (1)

$0 = (28.8D_1 + 21.6D_2 - 28.8D_5 - 21.6D_6)(10^6)$ (2)

$0 = (-75D_1 + 150D_3)(10^6)$ (3)

$0 = (100D_4 - 100D_6)(10^6)$ (4)

$0 = (-38.4D_1 - 28.8D_2 + 151.8D_5)(10^6) + [-0.75(10^6)]$ (5)

$0 = (-28.8D_1 - 21.6D_2 - 100D_4 + 143.2D_6)(10^6)$ (6)

$0 = (100D_7)(10^6)$ (7)

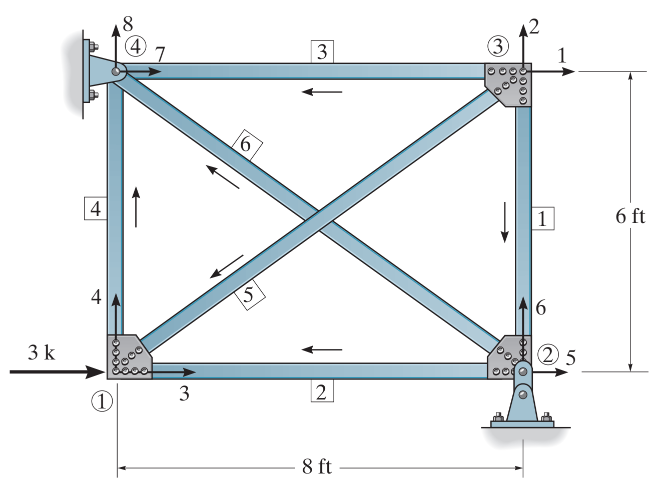
Solving Eqs. (1) to (7)

$D_1 = 0 \quad D_2 = 0.02667 \quad D_3 = 0 \quad D_4 = 0.01333$

$D_5 = 0.01 \quad D_6 = 0.01333 \quad D_7 = 0$

$D_6 = 0.0133$ m **Ans.**

***14–12.** Determine the stiffness matrix **K** for the truss. Take $A = 2$ in^2, $E = 29(10^3)$ ksi.



The origin of the global coordinate system is set at joint 1.

For member $\boxed{1}$, $L = 6(12) = 72$ in.,

$$\lambda_x = \frac{8 - 8}{6} = 0 \text{ and } \lambda_y = \frac{0 - 6}{6} = -1$$

$$\mathbf{k}_1 = \frac{2[29(10^3)]}{72} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 6 \end{array}$$

14–12. Continued

$$= \begin{array}{c} \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 805.56 & 0 & -805.56 \\ 0 & 0 & 0 & 0 \\ 0 & -805.56 & 0 & 805.56 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array}$$

For member $\boxed{2}$, $L = 8(12) = 96$ in., $\lambda_x = \dfrac{0-8}{8} = -1$ and $\lambda_y = \dfrac{0-0}{8} = 0$.

$$\mathbf{k}_2 = \frac{2[29(10^3)]}{96} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 5 \\ 6 \\ 3 \\ 4 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 604.17 & 0 & -604.17 & 0 \\ 0 & 0 & 0 & 0 \\ -604.17 & 0 & 604.17 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 5 \\ 6 \\ 3 \\ 4 \end{array}$$

For member $\boxed{3}$, $L = 8(12) = 96$ in., $\lambda_x = \dfrac{0-8}{8} = -1$ and $\lambda_y = \dfrac{6-6}{8} = 0$.

$$\mathbf{k}_3 = \frac{2[29(10^3)]}{96} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \end{array}$$

$$= \begin{array}{c} \begin{array}{cccc} 1 & 2 & 7 & 8 \end{array} \\ \begin{bmatrix} 604.17 & 0 & -604.17 & 0 \\ 0 & 0 & 0 & 0 \\ -604.17 & 0 & 604.17 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \end{array}$$

For member $\boxed{4}$, $L = 6(12) = 72$ in., $\lambda_x = \dfrac{0-0}{6} = 0$, and $\lambda_y = \dfrac{6-0}{6} = 1$

$$\mathbf{k}_4 = \frac{2[29(10^3)]}{72} \begin{array}{c} \begin{array}{cccc} 3 & 4 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \\ 7 \\ 8 \end{array}$$

14–12. Continued

$$= \begin{array}{c} \begin{matrix} 3 & 4 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 805.56 & 0 & -805.56 \\ 0 & 0 & 0 & 0 \\ 0 & -805.56 & 0 & 805.56 \end{bmatrix} \end{array} \begin{matrix} 3 \\ 4 \\ 7 \\ 8 \end{matrix}$$

For member $\boxed{5}$, $L = 10(12) = 120$ in., $\lambda_x = \dfrac{0-8}{10} = -0.8$ and $\lambda_y = \dfrac{0-6}{10} = -0.6$.

$$\mathbf{k}_5 = \frac{2[29(10^3)]}{120} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$= \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 309.33 & 232 & -309.33 & -232 \\ 232 & 174 & -232 & -174 \\ -309.33 & -232 & 309.33 & 232 \\ -232 & -174 & 232 & 174 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

For member $\boxed{6}$, $L = 10(12) = 120$ in., $\lambda_x = \dfrac{0-8}{10} = -0.8$ and $\lambda_y = \dfrac{6-0}{10} = 0.6$.

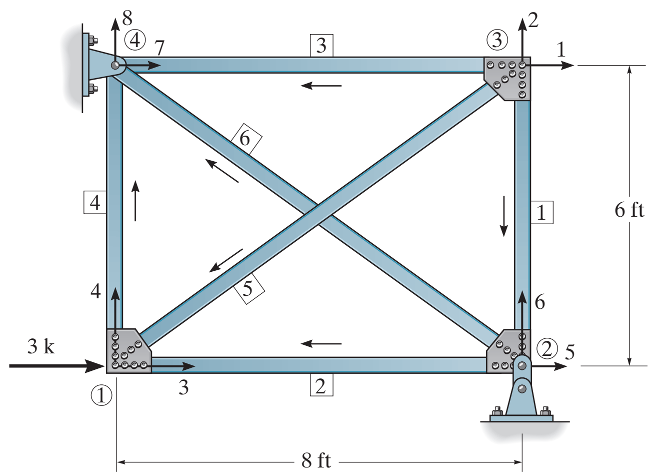
$$\mathbf{k}_6 = \frac{2[29(10^3)]}{120} \begin{array}{c} \begin{matrix} 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix} \end{array} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

$$= \begin{array}{c} \begin{matrix} 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 309.33 & -232 & -309.33 & 232 \\ -232 & 174 & 232 & -174 \\ -309.33 & 232 & 309.33 & -232 \\ 232 & -174 & -232 & 174 \end{bmatrix} \end{array} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

The structure stiffness matrix is a 8×8 matrix since the highest code number is 8. Thus,

$$\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 913.5 & 232 & -309.33 & -232 & 0 & 0 & -604.17 & 0 \\ 232 & 979.56 & -232 & -174 & 0 & -805.56 & 0 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 & 0 & 0 & 0 \\ -232 & -174 & 232 & 979.56 & 0 & 0 & 0 & -805.56 \\ 0 & 0 & -604.17 & 0 & 913.5 & -232 & -309.33 & 232 \\ 0 & -805.66 & 0 & 0 & -232 & 979.56 & 232 & -174 \\ -604.17 & 0 & 0 & 0 & -309.33 & 232 & 913.5 & -232 \\ 0 & 0 & 0 & -805.56 & 232 & -174 & -232 & 979.56 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

14–13. Determine the horizontal displacement of joint ② and the force in member $\boxed{5}$. Take $A = 2\text{ in}^2$, $E = 29(10^3)$ ksi. Neglect the short link at ②.



Here,

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \qquad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 6 \\ 7 \\ 8 \end{matrix}$$

Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \left[\begin{array}{ccccc|ccc} 913.5 & 232 & -309.33 & -232 & 0 & 0 & -604.17 & 0 \\ 232 & 979.56 & -232 & -174 & 0 & -805.56 & 0 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 & 0 & 0 & 0 \\ -232 & -174 & 232 & 979.56 & 0 & 0 & 0 & -805.56 \\ 0 & 0 & -604.17 & 0 & 913.5 & -232 & -309.33 & 232 \\ \hline 0 & -805.56 & 0 & 0 & -232 & 979.56 & 232 & -174 \\ -604.17 & 0 & 0 & 0 & -309.33 & 232 & 913.5 & -232 \\ 0 & 0 & 0 & -805.56 & 232 & -174 & -232 & 979.56 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition; $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 913.5 & 232 & -309.33 & -232 & 0 \\ 232 & 979.56 & -232 & -174 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 \\ -232 & -174 & 232 & 979.56 & 0 \\ 0 & 0 & -604.17 & 0 & 913.5 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = 913.5D_1 + 232D_2 - 309.33D_3 - 232D_4 \qquad (1)$$

$$0 = 232D_1 + 979.59D_2 - 232D_3 - 174D_4 \qquad (2)$$

$$3 = -309.33D_1 - 232D_2 + 913.5D_3 + 232D_4 - 604.17D_5 \qquad (3)$$

$$0 = -232D_1 - 174D_2 + 232D_3 + 979.56D_4 \qquad (4)$$

$$0 = -604.17D_3 + 913.5D_5 \qquad (5)$$

Solving Eqs. (1) to (5),

$D_1 = 0.002172 \qquad D_2 = 0.001222 \qquad D_3 = 0.008248 \qquad D_4 = -0.001222$

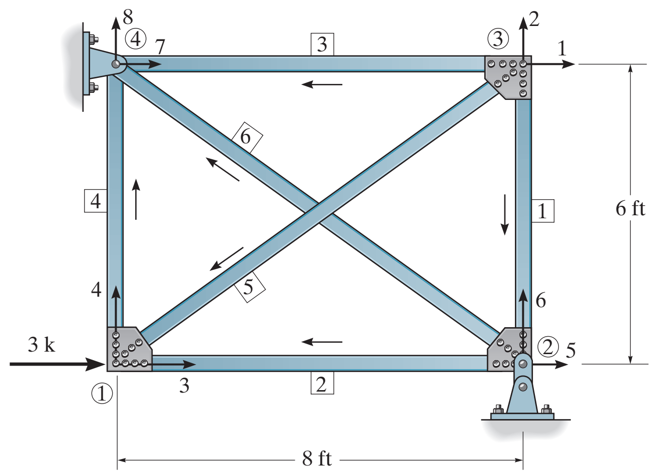
$D_5 = 0.005455 = 0.00546$ m **Ans.**

Force in Member $\boxed{5}$. Here, $L = 10(12) = 120$ in., $\lambda_x = -0.8$ and $\lambda_y = -0.6$

$$(q_5)_F = \frac{2[29(10^3)]}{120} \begin{bmatrix} 0.8 & 0.6 & -0.8 & -0.6 \end{bmatrix} \begin{bmatrix} 0.002172 \\ 0.001222 \\ 0.008248 \\ -0.001222 \end{bmatrix} \begin{matrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{matrix}$$

$= 1.64$ k (C) **Ans.**

14–14. Determine the force in member $\boxed{3}$ if this member was 0.025 in. too short before it was fitted onto the truss. Take $A = 2\text{ in}^2$. $E = 29(10^3)$ ksi. Neglect the short link at ②.



For member $\boxed{3}$, $L = 8(12) = 96$ in $\lambda_x = -1$, $\lambda_y = 0$ and $\Delta L = -0.025$. Thus,

$$\begin{bmatrix} (Q_1)_0 \\ (Q_2)_0 \\ (Q_7)_0 \\ (Q_8)_0 \end{bmatrix} = \frac{2[29(10^3)](-0.025)}{96}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 15.10 \\ 0 \\ -15.10 \\ 0 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 7 \\ 8 \end{array}$$

Also,

$$\mathrm{Q}_k = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \quad \text{and} \quad \mathrm{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} 6 \\ 7 \\ 8 \end{array}$$

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = \begin{bmatrix} 913.5 & 232 & -309.33 & -232 & 0 & 0 & -604.17 & 0 \\ 232 & 979.56 & -232 & -174 & 0 & -805.56 & 0 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 & 0 & 0 & 0 \\ -232 & -174 & 232 & 979.56 & 0 & 0 & 0 & -805.56 \\ 0 & 0 & -604.17 & 0 & 913.5 & -232 & -309.33 & 232 \\ 0 & -805.56 & 0 & 0 & -232 & 979.56 & 232 & -174 \\ -604.17 & 0 & 0 & 0 & -309.33 & 232 & 913.5 & -232 \\ 0 & 0 & 0 & -805.56 & 232 & -174 & -232 & -979.56 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 15.10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -15.10 \\ 0 \end{bmatrix}$$

Applying $\mathbf{Q} = \mathbf{KD} + \mathbf{Q}_0$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k + (\mathbf{Q}_k)_0$,

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 913.5 & 232 & -309.33 & -232 & 0 \\ 232 & 979.56 & -232 & -174 & 0 \\ -309.33 & -232 & 913.5 & 232 & -604.17 \\ -232 & -174 & 232 & 979.56 & 0 \\ 0 & 0 & -604.17 & 0 & 913.5 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 15.10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this matrix equality,

$$0 = 913.5D_1 + 232D_2 - 309.33D_3 - 232D_4 + 15.10 \qquad (1)$$

$$0 = 232D_1 + 979.56D_2 - 232D_3 - 174D_4 \qquad (2)$$

$$3 = -309.33D_1 - 232D_2 + 913.5D_3 + 232D_4 - 604.17D_5 \qquad (3)$$

$$0 = -232D_1 - 174D_2 + 232D_3 + 979.56D_4 \qquad (4)$$

$$0 = -604.17D_3 + 913.5D_5 \qquad (5)$$

Solving Eqs. (1) to (5),

$D_1 = -0.01912 \quad D_2 = 0.003305 \quad D_3 = -0.002687 \quad D_4 = -0.003305$

$D_5 = -0.001779$

14–14. Continued

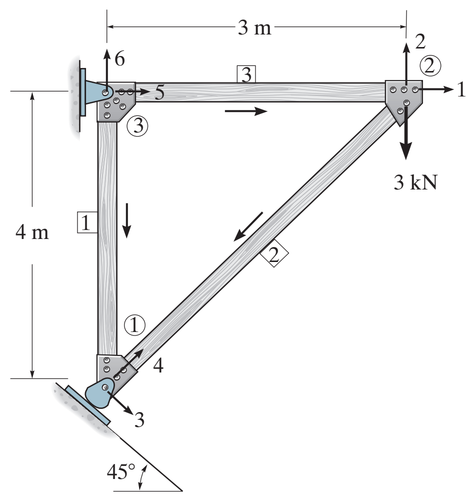
Force in member $\boxed{3}$. Here, $L = 8(12) = 96$ in., $\lambda_x = -1$, $\lambda_y = 0$ and

$$(q_F)_0 = \frac{-2[29(10^3)]\,(-0.025)}{96} = 15.10 \text{ k}$$

$$(q_3)_F = \frac{2[29(10^3)]}{96} \quad [1 \quad 0 \quad -1 \quad 0 \begin{bmatrix} -0.01912 \\ 0.003305 \\ 0 \\ 0 \end{bmatrix} + 15.10$$

$$= 3.55 \text{ k (T)}$$ **Ans.**

14–15. Determine the stiffness matrix **K** for the truss. AE is constant.

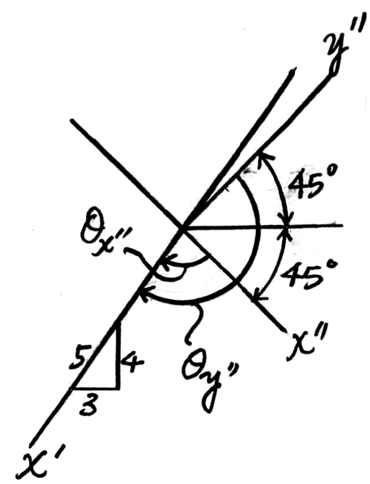


The origin of the global coordinate system is set at joint ①.

For member $\boxed{2}$, $L = 5$ m. Referring to Fig. a, $\theta''_x = 180° - 45° - \sin^{-1}\left(\frac{4}{5}\right) = 81.87°$

$\theta''_y = 171.87°$. Thus, $\chi''_x = \cos\theta_{x''} = \cos 81.87° = 0.14142$ and

$\lambda_{y''} = \cos\theta_{y''} = \cos 171.87° = -0.98995$



(a)

Also, $\lambda_x = \dfrac{0-3}{5} = -0.6$ and $\lambda_y = \dfrac{0-4}{5} = -0.8$

$$\mathbf{k}_1 = AE \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.072 & 0.096 & 0.01697 & -0.11879 \\ 0.096 & 0.128 & 0.02263 & -0.15839 \\ 0.01697 & 0.02263 & 0.004 & -0.028 \\ -0.11879 & -0.15839 & -0.028 & 0.196 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

For member $\boxed{1}$, $L = 4$ m. Referring to Fig. b, $\theta_{x''} = 45°$ and $\theta_{y''} = 135°$.

Thus, $\lambda_{x''} = \cos 45° = \dfrac{\sqrt{2}}{2}$ and $\lambda_{y''} = \cos 135° = -\dfrac{\sqrt{2}}{2}$.

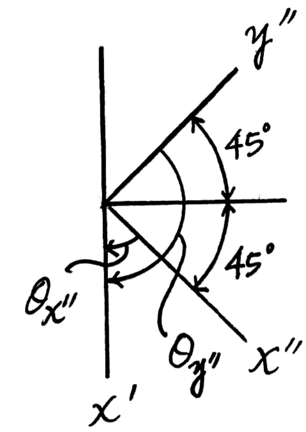
Also, $\lambda_x = 0$ and $\lambda_y = -1$.

$$\mathbf{k}_2 = AE \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.17678 & -0.17678 \\ 0 & 0.17678 & 0.125 & -0.125 \\ 0 & -0.17678 & -0.125 & 0.125 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$

For member $\boxed{3}$, $L = 3$ m, $\lambda_x = 1$ and $\lambda_y = 0$.

$$\mathbf{k}_3 = AE \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.33333 & 0 & -0.33333 & 0 \\ 0 & 0 & 0 & 0 \\ -0.33333 & 0 & 0.33333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 2 \end{array}$$

14–15. Continued

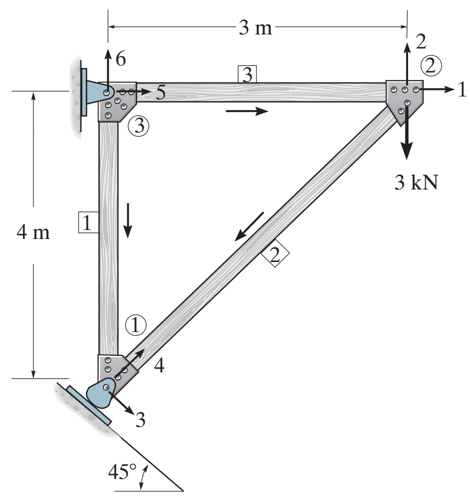


(b)

The structure stiffness matrix is a 6 × 6 matrix since the highest code number is 6. Thus,

$$\mathbf{k} = AE \begin{array}{c} \begin{array}{cccccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad & \quad 5 \quad & \quad 6 \quad \end{array} \\ \left[\begin{array}{cccccc} 0.40533 & 0.096 & 0.01697 & -0.11879 & -0.33333 & 0 \\ 0.096 & 0.128 & 0.02263 & -0.15839 & 0 & 0 \\ 0.01697 & 0.02263 & 0.129 & -0.153 & 0 & 0.17678 \\ -0.11879 & -0.15839 & -0.153 & 0.321 & 0 & -0.17678 \\ -0.33333 & 0 & 0 & 0 & 0.33333 & 0 \\ 0 & 0 & 0.17678 & -0.17678 & 0 & 0.25 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

***14–16.** Determine the vertical displacement of joint ② and the support reactions. AE is constant.



Here,

$$\mathbf{Q}_k = \left[\begin{array}{c} 0 \\ -3(10^3) \\ 0 \end{array}\right] \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \text{ and } \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} 4 \\ 5 \\ 6 \end{array}$$

Applying $\mathbf{Q} = \mathbf{KD}$

$$\left[\begin{array}{c} 0 \\ -3(10^3) \\ 0 \\ \hline Q_4 \\ Q_5 \\ Q_6 \end{array}\right] = AE \left[\begin{array}{ccc|ccc} 0.40533 & 0.096 & 0.01697 & -0.11879 & -0.33333 & 0 \\ 0.096 & 0.128 & 0.02263 & -0.15839 & 0 & 0 \\ 0.01697 & 0.02263 & 0.129 & -0.153 & 0 & 0.17678 \\ \hline -0.11879 & -0.15839 & -0.153 & 0.321 & 0 & -0.17678 \\ -0.33333 & 0 & 0 & 0 & 0.33333 & 0 \\ 0 & 0 & 0.17678 & -0.17678 & 0 & 0.25 \end{array}\right] \left[\begin{array}{c} D_1 \\ D_2 \\ D_3 \\ \hline 0 \\ 0 \\ 0 \end{array}\right]$$

From the matrix partition; $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$\left[\begin{array}{c} 0 \\ -3(10^3) \\ 0 \end{array}\right] = AE \left[\begin{array}{ccc} 0.40533 & 0.096 & 0.01697 \\ 0.096 & 0.128 & 0.02263 \\ 0.011697 & 0.02263 & 0.129 \end{array}\right] \left[\begin{array}{c} D_1 \\ D_2 \\ D_3 \end{array}\right] + \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right]$$

Expanding this matrix equality,

$$0 = AE(0.40533\, D_1 + 0.096\, D_2 + 0.01697\, D_3) \qquad (1)$$

$$-3(10^3) = AE(0.096\, D_1 + 0.128\, D_2 + 0.02263\, D_3) \qquad (2)$$

$$0 = AE(0.01697\, D_1 + 0.02263\, D_2 + 0.0129\, D_3) \qquad (3)$$

14–16. Continued

Solving Eqs. (1) to (3),

$$D_1 = \frac{6.750(10^3)}{AE} \quad D_3 = \frac{4.2466(10^3)}{AE}$$

$$D_2 = \frac{-29.250(10^3)}{AE} = \frac{29.3(10^3)}{AE} \quad \downarrow$$ **Ans.**

Again, the matrix partition $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$ gives

$$\begin{bmatrix} Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = AE \begin{bmatrix} -0.11879 & -0.15839 & -0.153 \\ -0.33333 & 0 & 0 \\ 0 & 0 & 0.17678 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 6.750(10^3) \\ -29.250(10^3) \\ 4.2466(10^3) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$Q_4 = 3.182(10^3)\text{ N} = 3.18\text{ kN} \qquad Q_5 = -2.250(10^3)\text{ N} = -2.25\text{ kN}$ **Ans.**

$Q_6 = 750\text{ N}$ **Ans.**